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SUPPLEMENTARY NORMAL ACCELERATION a_n*

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SUPPLEMENTARY NORMAL ACCELERATION a_n*

KINEMATIC A ND DYNAMIC MEANING OF ANGULAR VELOCITY ω*

1. At first, we begin with the study of the trajectory of a material point from the kinematic point of view exclusively. In classical kinematics a differential ***ds*** of arc in the trajectory is substituted by the corresponding in the osculating circle in order to calculate the acceleration vector. For this purpose a FRENET's refferential frame is used. The acceleration components in this circle are

$$\boldsymbol{a}_s = (dv/dt)\boldsymbol{s} \qquad \text{and} \qquad \boldsymbol{a}_\rho = -(v^2/\rho))\boldsymbol{n} \qquad (1)$$

And where ***s*** and ***n*** are the *versors,* in this frame: ***s*** , ***n*** , ***b*** , whose positive sense in the *tangent, normal* and *binormal,* is determined by the velocity sense, by the sense towards convexity and by the vectorial product: $\boldsymbol{b} = \boldsymbol{s} \times \boldsymbol{n}$, respectively. The angular velocity is

$$\boldsymbol{\omega} = (v/\rho))\boldsymbol{b}$$

A definite trajectory has a well defined *evolute,* and in the calculation of the normal component in the expressions (1) the differentials dv and $d\rho$ are obviously not taken into account. But, as we will demonstrate, when $dv \neq 0$ and $d\rho \neq 0$, the *arc* of the *evolute* does not correspond with the real one: it turns locally at an angular velocity

$$\boldsymbol{\omega}* = (dv/d\rho))\boldsymbol{b}$$

and the same thing occurs with the corresponding *arc* of *trajectory* in the osculating circle.

In order to explain the kinematic meaning of this angular velocity $\boldsymbol{\omega}*$, we shall study an element $\boldsymbol{ds}$ of trajectory which corresponds to the $\boldsymbol{d\rho}$ of the *evolute;* they are both located on the plane of osculation (see Fig.1 when $dv/dt > 0$; and Fig. 2 when $dv/dt < 0$). Thus we can consider the trajectory as being locally plane and referred to an intrinsic frame with *versors* $\boldsymbol{s}$, $\boldsymbol{n}$, $\boldsymbol{b}$, formed by the *tangent, normal* and the *binormal.* The *arc* $\boldsymbol{ds}$ of the trajectory is determined by the points A , B , and the $\boldsymbol{d\rho}$, of the *evolute,* on account of its equivalent points A ,B . The speed of the particle in A is v and in B it is $v + dv$. The radii of curvature at these points are: $\rho + d\rho$ and ρ . The angle turned by the radius of curvature when it passes from A to B is

$$d\theta = ds/\rho$$

and the corresponding angular speed will be as we have seen

$$\omega = d\theta/dt \qquad (\text{with} \quad \boldsymbol{\omega} = \omega \boldsymbol{b})$$

We can also write: $\omega = v/\rho$, which evidently does not depend on dv and $d\rho$. When we calculate the centripetal acceleration we get the expression (1) in the form

$$\boldsymbol{a}_\rho = (-v^2/\rho))\boldsymbol{n}$$

in which the increases dv , $d\rho$, are not considered, as they do not affect it. It is the consequence of replacing the $\boldsymbol{ds}$ of *trajectory* by corresponding one in the *osculating circle* at the same point. However, if we observe the real trajectory carefully, we see that is characterized by having a well determined *evolute* (see Fig. 1, when $dv/dt > 0$, and Fig. 2, when $dv/dt < 0$). When dv is dispensed with, in the study of centripetal acceleration, it means that starting out from point A we arrive at B' but not at the real point B ; and the same must also be said of its equivalent centre of curvature: A is located in the *evolute,* as it is the starting point, but B' lay outside of the *real evolute* (see Fig.1 and 2), whose point is B . It is evident that the centripetal acceleration is correctly determined, but it is also clear that the *arc* of the *evolute* must coincide with what is determined by points A and B in the figure, and not by the A and B', as happens when dv and $d\rho$ are omitted. In order to rectify this deficiency it is necessary to rotate AB' an angle

$$d\theta^* = BB'/d\rho$$

so that it coincides with the $\boldsymbol{d\rho}$ in the *evolute,* with a *finite* angular velocity (see Fig. 1 and Fig. 2) whose module is expressed by

$$(BB'/d\rho)/dt = (d^2s/d\rho)/dt = dv/d\rho = d\theta^*/dt = \omega^*$$

This angular velocity shows that the simplification of replacing the trajectory with the osculating circle in each point means that it is necessary to turn locally the *arc* of the *evolute*, with angular velocity $\boldsymbol{\omega^*}$, so that it coincides with the real one. But this *arc* AB' of the *evolute* must be *normal* to the corresponding AB'' of the *trajectory,* rotated also $d\theta^*$, with respect to the initial AB (see Fig. 1 and Fig. 2). It will be necessary to turn AB' this angle, in the *same sense* (when $dv/dt > 0$) and in the *opposite sense* (when $dv/dt < 0$), so that it coincides with the *real* one . As a result, the *radius* ρ has increased in a second order infinitesimal amount:

$$B'B'' = ds d\theta \qquad \text{(when } dv/dt > 0)$$

and

$$B'B'' = -ds d\theta \qquad \text{(when } dv/dt < 0)$$

and the immediate result is a *supplementary normal acceleration:*

$$\alpha_\rho^* = B'B''/dt^2 = ds\, d\theta^*/dt^2 = v\omega^* \quad \text{(when } dv/dt > 0\text{)}$$

$$\alpha_\rho^* = B'B''/dt^2 = -\, ds\, d\theta^*/dt^2 = -\, v\omega^* \quad \text{(when } dv/dt < 0\text{)}$$

superimposed to the *normal acceleration* a_ρ (1). So the *total normal acceleration* is

$$a_\rho + a +_\rho^* = -(v\omega + v\omega^*) = -v(\omega - \omega^*)$$

$$a_\rho + a +_\rho^* = -(v\omega - v\omega^*) = -v(\omega + \omega^*)$$

in the two possible cases.

Obviusly the *tangential acceleration* $a_s = dv/dt$ remains unchanged. In vectorial form we get

$$a_s\mathbf{s} + a_\rho\mathbf{n} + a_\rho^*\mathbf{n} = \mathbf{a} + v\omega^*\mathbf{n} = \mathbf{a} - \mathbf{v}\times\boldsymbol{\omega}^*$$

$$a_s\mathbf{s} + a_\rho\mathbf{n} + a_\rho^*\mathbf{n} = \mathbf{a} - v\omega^*\mathbf{n} = \mathbf{a} + \mathbf{v}\times\boldsymbol{\omega}^* \qquad (2)$$

respectively.

2. Now, from the dynamical point of view, if we want to calculate the *total centripetal force* correctly, the *total normal acceleration* (16) must be taken into account. So the expression of this force will be

$$\mathbf{f_n} = -mv\,(\omega - \omega^*)\mathbf{n} = m\mathbf{v} \times (\boldsymbol{\omega} - \boldsymbol{\omega}^*)$$

and

$$\mathbf{f_n} = -mv\,(\omega + \omega^*)\mathbf{n} = m\mathbf{v} \times (\boldsymbol{\omega} + \boldsymbol{\omega}^*) \qquad (3)$$

in both possible cases.

In summary, taken in account the expression (2), the *total force* acting on the material point is

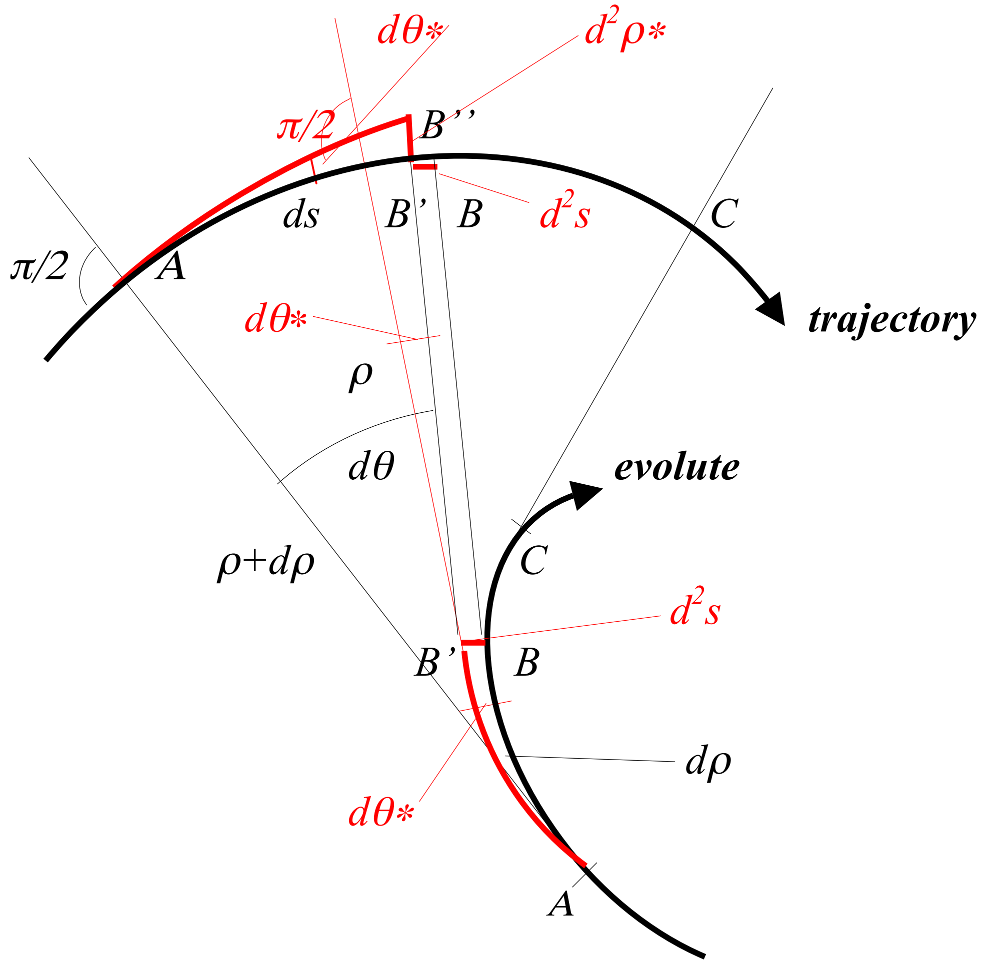
$$\boldsymbol{f} = m(\boldsymbol{a} \pm \boldsymbol{v} \times \boldsymbol{\omega}*) \qquad (4)$$

(which is *isomorphic* with the LORENTZ electromagnetic force).

The angular velocity $\boldsymbol{\omega}*$ will only cease to exist when the trajectory is a circumference or the speed v is constant, as it follows observing Fig.1 and Fig. 2 .(see also the cases of Fig 1' and Fig.2').

The result (4) is surprising: even more so when we remember that "LORENTZ's force" is exclusively experimental. Moreover, in FRENET's trihedron the value v of speed is always *positive* in the sense in which the particle is moving . We know that while the moving point follows the trajectory, the *centre of curvature,* at the corresponding point, describes the *evolute*, and we can take the sign of $d\rho$ as *positive* because the sense of its movement follows the *changing sense* of the velocity $\boldsymbol{v}$. This result is of the major importance (see the two possible cases in Figs. 1, 2', y 1', 2) because $\omega = dv/d\rho$ *changes sign,* when the movement is inverted (*dv changes to $-dv$* whereas $d\rho$, in the *evolute does not change).* When the *movement is inverted,* the *versor* $\mathbf{s}\times \boldsymbol{b} = -\boldsymbol{n}$ *manteins its sense,* but the *supplementary acceleration* $\boldsymbol{a}* = \boldsymbol{v}\times \boldsymbol{\omega}* = \mathbf{s}\times \boldsymbol{b}v\omega$ *changes it* when $\omega*$ *changes to* $-\omega*$. Consequently, the ***reversibility*** of the trajectory in CD ***does not hold up*** in the ND,

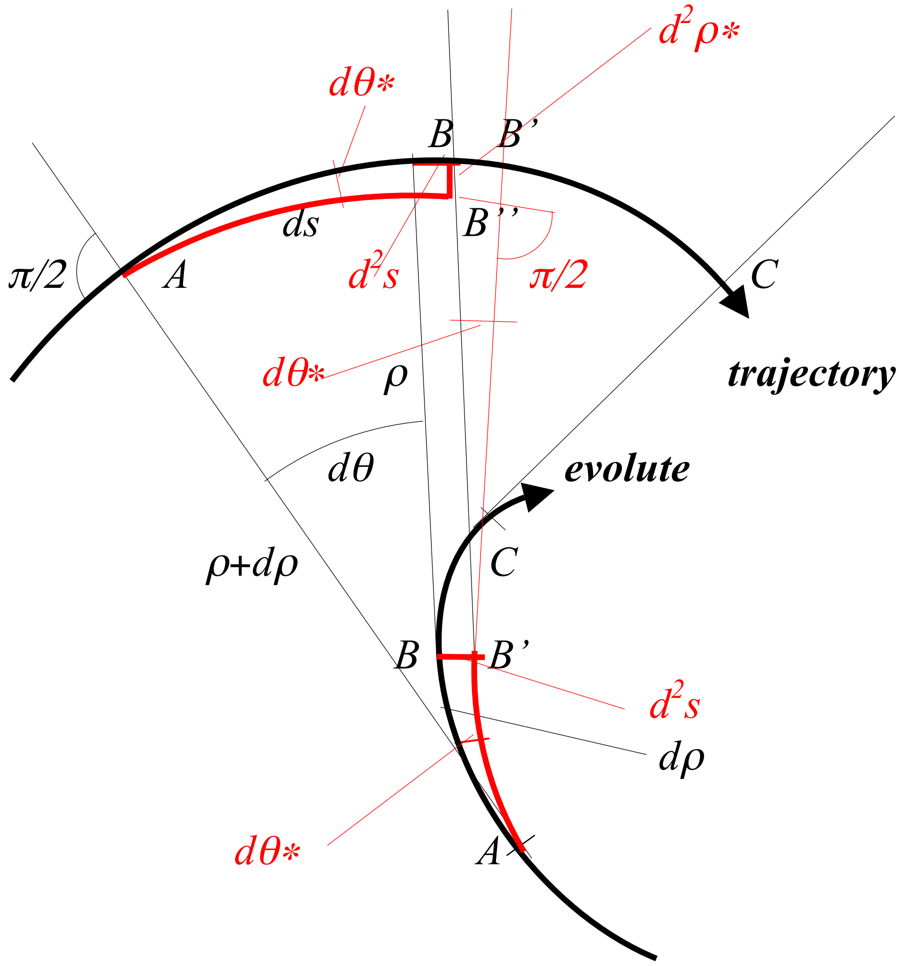
The ***CHAOS*** presence in physical phenomena has its foundation in this ***irreversibility.***



Supplementary Normal Acceleration (when *dv/dt < 0)*

$$a_n{*} \;=\; d^2\rho{*}/dt^2$$

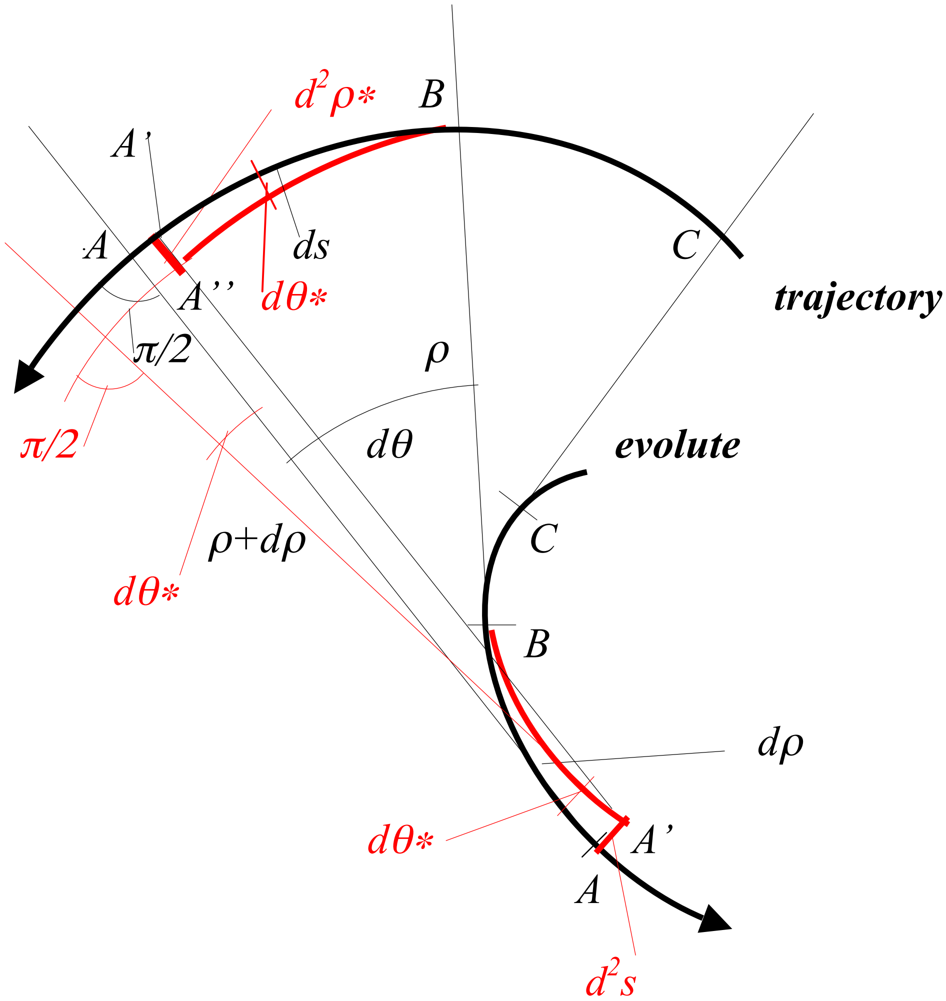
FIG. 1



Supplementary Normal Acceleration (when *dv/dt > 0)*

$$a_n{*} \;=\; d^2\rho{*}/dt^2$$

FIG. 2

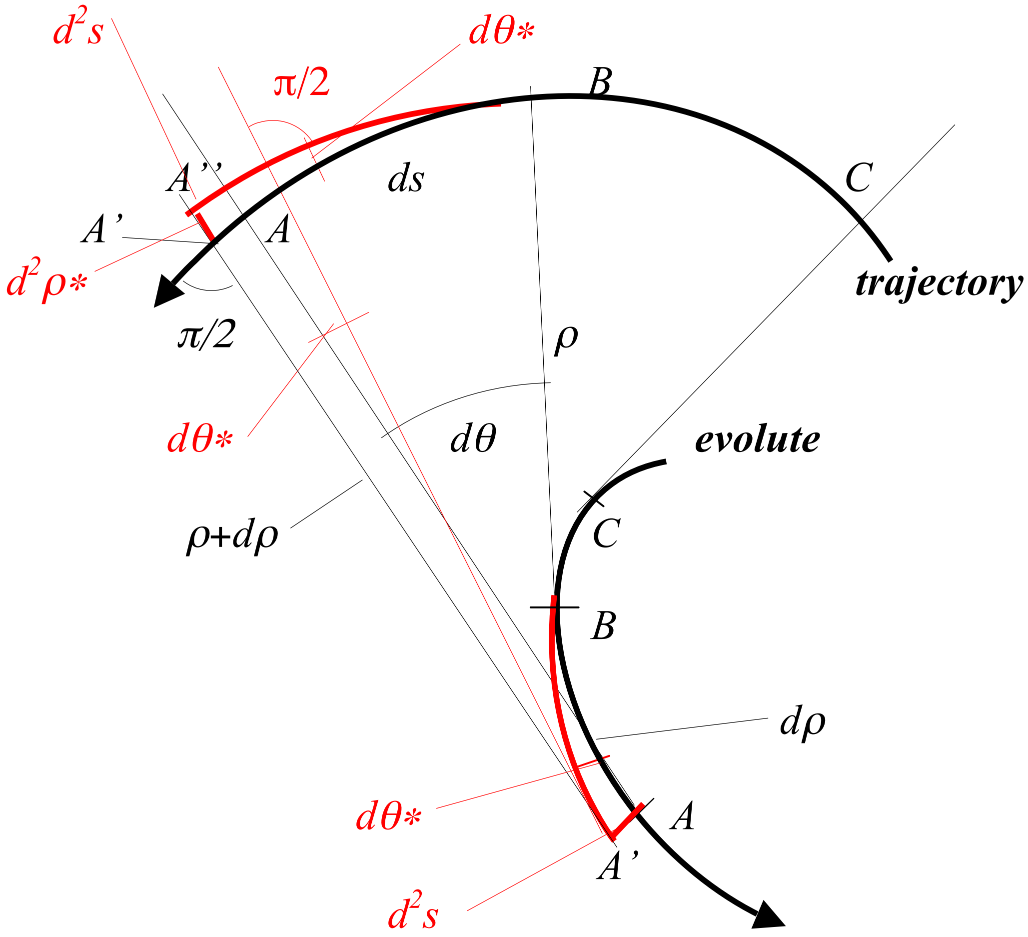


Supplementary Normal Acceleration

(running in *inverse direction*, with *dv/dt < 0)*

$$a_n* = d^2\rho*/dt^2$$

FIG. 1'



Supplementary Normal Acceleration
(running in *inverse direction*, with $dv/dt > 0$)

$$a_n* = d^2\rho*/dt^2$$

FIG. 2'

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